A NEW PROBABILISTIC METHOD FOR FORECASTING MORTALITY: A CASE STUDY OF ESTONIA

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Abstract

A new probabilistic mortality forecasting approach is introduced that, unlike the Lee-Carter Method (and its variants), is directly linked to the fundamental demographic equation, the cornerstone of demographic theory. This is an important consideration in developing accurate forecasts. Because it forecasts "years lived," this new approach directly yields life expectancy and a corresponding future life table (examples are provided in the Appendix, along with a description of the steps needed to construct them), unlike the Lee-Carter Method and its variants. In an ex post facto evaluation using Estonian data from the Human Mortality Database, the new approach was found to provide accurate forecasts of "years lived by age" (${}_nL_x$) for both point and interval measures over 15 years. In a broader context, the accuracy of Estonia's point ${}_nL_x$ forecasts is compared with that of Algeria, Australia, Japan, and the United States. Probabilistic ${}_nL_x$ forecasts for Estonia from 2029 to 2039 are then provided, the results are discussed, and the next steps for evaluating this approach are suggested.

Background

Mortality forecasting is an important activity (Bengtsson and Keilman, 2019). It is used in the preparation of population forecasts based on the cohort component (CCM) method (Smith, Tayman, and Swanson, 2013: 61-72), the development of social welfare, annuity and pension products (Booth and Tickle, 2008; Haberman and Renshaw, 2011; Huang, Maller, and Ning, 2020; Lee and Miller, 2001; Rabbi and Mazzuco, 2020; Shang, Booth, and Hyndman, 2011; Tableau, Van Den Berg Jeths, and Heathcote, 2001) and epidemiological/health research (Andrade, Camarda, and Arolas, 2025; Booth and Tickle, 2008; Swanson, Bryan, and Chow, 2020).

The Lee-Carter approach to forecasting mortality was introduced in 1992 (Lee and Carter, 1992) and, along with its refinements and variants, is arguably the most widely used approach in the world (Andreozzi et al., 2011; Basellini, Camarda, and Booth, 2023; Booth and Tickle, 2008; Rabbi and Mazzuco, 2020). As observed by Basellini, Camarda, and Booth (2023: 1034), it is useful to examine the Lee-Carter approach in terms of two aspects: the model and the method. The model is a functional form for age-specific mortality (age-specific death rates, ASDRs), and the method consists of a series of steps to estimate the model and fit a time-series model to the time index, along with specific adjustment and estimation procedures. Both the model and the method are essentially mathematical fitting procedures and have no direct relationship to the dynamics of population change or its components, one of which is obviously mortality. Moreover, to generate a forecast of life expectancy and a corresponding life table, the Lee-Carter Method and its variants require that the ASDRs be converted into a life table (e.g., Fergany's method (Fergany, 1971) and the Keyfitz-Frauenthal method (Kintner, 2004: 314-315)).

This paper introduces a new method of mortality forecasting by showing how measures of uncertainty from a standard time series model, "Auto Regressive Integrated Moving Average"

("ARIMA," Box and Jenkins, 1976), can be applied to a population forecast based on the Hamilton-Perry Method ("H-P," Baker et al., 2017) that generates "years lived" by age (${}_nL_x$) and which also includes Total years lived (T_0) and life expectancy at birth (e_0) as found in an abridged period life table. From the perspective of formal demography, this is a forecast of the age distribution and size of the stationary population, given the mortality and age structure of a given population (see, e.g., Ryder, 1975; Rao and Carey, 2015; Swanson and Tedrow, 2021). Appendix A shows how a forecasted period life table and a probabilistic life expectancy forecast can be constructed from this approach to forecasting ${}_nL_x$.

The measures of forecast uncertainty are relatively easy to calculate and meet several important criteria used by demographers who routinely generate forecasts, including utility (Tayman and Swanson, 1996) as well as face validity, plausibility, production cost, timeliness, ease of application and ease of explanation (McNown, Rogers, and Little, 1995; Smith, Tayman, and Swanson, 2013: 302-315). Unlike the Lee-Carter method and its corresponding "principal components" variants (Booth and Tickle, 2008; Lee and Carter, 1992; Lee and Miller, 2001; Shang, Booth, and Hyndman, 2011), this approach, given three major constraints (described later), links the probabilistic forecast uncertainty to the fundamental demographic equation, the cornerstone of demographic theory. In addition to being a potential contribution to formal demography, this is an important consideration in developing accurate forecasts (Swanson et al., 2023). Also, unlike the Lee-Carter method and its variants, this new approach directly yields life expectancy and a corresponding future life table because it directly forecasts "years lived" by age (${}_nL_x$). An ex post facto evaluation of the method's accuracy is conducted as a case study using Estonian data from the Human Mortality Database (HMD), and an example forecast set using current data is provided for Estonia.

Data

We selected Estonia for this case study mainly for two reasons. First, its population is small: As of 1 January 2025, Statistics Estonia (2026) shows it as 1,369,995. We wanted a small population (which likely would have more stochastic variation in a corresponding set of ${}_nL_x$ values than found in a large population) in this case study because our experience in working with large and small populations suggested that if the evaluation of our proposed method shows that it works well in a small population, it is, with some caveats (found in the "Evaluation" and "Discussion" sections), likely to work well not only in other small populations but also in large populations. Second, its data are of high quality and found in HMD (2025), which is where we obtained annual ${}_nL_x$ data from 1959 to 2024 that were organized in such a manner that made it easy to assemble into 18 age groups (0-4, 5-9, 10-14, …,75-79, 80-84, and 85+). We also computed the ratio ${}_nL_x$ / 42,388, where 42,388 is the land area of Estonia (km2), yielding annual ${}_nL_x$ "density" values for each of the 18 age groups from 1959 to 2024. We discuss why we used density values in the "Transferring Uncertainty" section.

Methods

We employ the Hamilton-Perry (H-P) method (Baker et al., 2017), which computes cohort change ratios (CCRs) using two counts of the age structure (${}_nL_x$) in question, typically five or ten years apart, which directly capture age-specific population dynamics. Before turning to a discussion of the probabilistic approach we use (which is followed by a description of our input data and the forecast results), it is helpful to note that the H-P method is algebraically equivalent to the fundamental demographic equation and therefore grounded in demographic theory (Baker et al., 2017: 251-252). Barring unforeseeable catastrophes and other events that have very low probabilities of occurring (Taleb, 2010), the closer one comes to having accurate data embedded in a method that is grounded in demographic theory, the more accurate a population forecast method will likely be (Swanson et al., 2023), a dictum that one could reasonably expect to apply to forecasting ${}_nL_x$.

Components of Demographic Change

There are three *components of population change*: mortality, fertility, and migration. The overall growth or decline of a population is determined by the interplay among these three components. The exact nature of this interplay can be formalized in the *fundamental demographic equation*:

$$P_l - P_b = B - D + IM - OM, \qquad [1]$$

where P_l is the population at the end of the time period; P_b is the population at the beginning of the time period; and B, D, IM, and OM are the number of births, deaths, in-migrants, and out-migrants during the time period, respectively. The difference between births and deaths is called natural change (B – D); it represents population growth from within the population itself. It may be either positive or negative, depending on whether births exceed deaths or deaths exceed births. The difference between the number of in-migrants and the number of out-migrants is called *net migration* (IM – OM); it represents population growth coming from the movement of people into and out of the area. It may be either positive or negative, depending on whether in-migrants exceed out-migrants or out-migrants exceed in-migrants. In cases where IM and OM do not occur (e.g., the world as a whole, the stationary population that is found in a life table), these elements can be omitted from the fundamental population equation.

The fundamental demographic equation can also be extended to apply to age groups, age-gender groups, age-gender-race groups, and age-gender-ethnicity groups. This type of extension provides the logical basis for the equation and can be used to project a population into the future by age,

age and gender, or age, gender, and race. Once launched, these components (which are frequently modified as the forecast moves forward based on assumptions about their direction) are applied to the resulting age-gender structure at each forecast cycle. In terms of ${}_nL_x$, there is no migration, eliminating the need for this component in forecasting ${}_nL_x$.

Hamilton-Perry Method

The Hamilton-Perry (H-P) method (Baker et al., 2017: 251-252; Tayman and Swanson, 2025) is typically used to forecast population by age and sex. It is also well-suited for generating a ${}_nL_x$ (person-years lived (PYL)) by age forecast per the framework found in Swanson et al. (2023): (1) It corresponds to the dynamics by which a population or PYL moves forward in time; (2) there is information available relevant to these dynamics; (3) the time and resources needed to assemble relevant information and generate a forecast are minimal; and (4) the information needed is generated by the H-P method. This method conforms to the fundamental population equation, but it does not apply the separate components of population change to the age structure at the launch year. Instead, the H-P method uses computes cohort change ratios (CCRs) using two counts of the age structure in question, typically five or ten years apart, which directly capture mortality and migration. The fertility component is reflected in the youngest age group, typically ages 0-4. Forecasting the 0-4 population uses a "child-adult ratio" from the most recent age structure data. We use a different ratio for the youngest population discussed below.

The H-P method moves the PYL by age from time t to time t+k (the forecast cycle length) using CCRs computed from data in the two most recent data points (e.g., censuses or estimates) with the proviso that the width of the age groups (other than the terminal, open-ended age group) can be divided into the length of the forecast cycle such that it yields a whole number as the quotient. It consists of two steps. The first uses existing data to develop CCRs, and the second applies the CCRs to the launch-year cohorts to forecast them into the future. The formula for the first step, the development of a CCR, is:

$$_{n}CCR_{x,i} = {}_{n}L_{x,i,t} / {}_{n}P_{x-k,i,t-k}, \quad [2]$$

where, $_{n}L_{x,i,t}$ is the PYL aged x to x+n in area i at the most recent census/estimate (t), $_{n}L_{x-k,i,t-k}$ is the population aged x-k to x-k+n in area i at the 2nd most recent census/estimate (t-k), and k is the number of years between the most recent census/estimate at time t for area i and the census/estimate preceding it for area i at time t-k.

The basic formula for the second step, moving the cohorts of a population into the future, is:

$$_{n}L_{x+k,i,t+k} = ({}_{n}CCR_{x,i}) \times ({}_{n}L_{x,i,t}), \quad [3]$$

where, $_{n}L_{x+k,i,t+k}$ is the PYL aged x+k to x+k+n in area i at time t+k.

Given the nature of the CCRs, they cannot be calculated for the youngest age group (i.e., ages 0-4 if it is a five-year forecast cycle; ages 0-9 if it is a ten-year forecast cycle), because this cohort came into existence after the census/estimate data collected at time t-k. Forecasting the PYL for the youngest age does not require any data beyond what is available in the census/estimate sets of successive data. In using the H-P method, the youngest age group is 5L0 (ages 0-4), as used in this paper. We obviously do not employ a CAR because the number of births in a life table is fixed, usually at 100,000 each year; thus, 5L0 comprises the survivors of these births. As such, we take the ratio of $_{5}L_{0}$ from the last and second most recent census or estimate. The equation for forecasting the PYL aged 0-4 is:

$$\text{PYL 0-4: } {}_{5}L_{0,i,t+k} = ({}_{5}L_{0,i,t} / {}_{5}L_{0,i,t-k}) \times ({}_{5}L_{0,i,t}) \quad [4]$$

where PYL is the person-years lived, t is the year of the most recent census, t-k is the year of the second most recent census or estimate, and t+k is the forecast year.

Forecasts of the oldest open-ended age group differ slightly from the H-P forecasts for the age groups beyond age 10 up to the oldest open-ended age group. If, for example, the final closed age group is 80-84, with 85+ as the terminal open-ended age group, then calculations for the $CCR_{i,x+}$ require the summation of the three oldest age groups to get the population age 75+ at time t-k in a ten-year forecast cycle (80+ in a five-year forecast cycle):

$$_{\infty}CCR_{75,i,t} = {}_{\infty}L_{85,i,t} / {}_{\infty}L_{75,i,t-k}. \qquad [5]$$

The formula for estimating the population of 85+ of area i for the year t+k is:

$$_{\infty}L_{85,i,t+k} = ({}_{\infty}CCR_{75,i,t}) \times ({}_{\infty}L_{75,i,t}). \qquad [6]$$

An issue that is found in the cohort change ratio for the terminal, open-ended age group (which in our case is 85 years and over) in a forecast where migration is not a component of population change is that like the equivalent probability of survival in an abridged life table, deaths are not uniformly distributed within the interval (Chiang, 1984: Lahiri, 2018; Swanson, Bryan, and Chow, 2020). This issue tends to exaggerate the length of life for those aged 85 and over in an abridged life table and in an H-P forecast.

Constraints

There are three major constraints in forecasting (or backcasting) $_nL_x$. These constraints affect any approach to forecasting $_nL_x$ based on the fundamental population equation. First, the births (and deaths) in a life table are typically set at 100,000 per year. Second, $_nL_x$ cannot exceed this constraint in that it is limited to n × 100,000 in a fixed-width age group of n years. Third, $_nL_{x,\,t+k}\,, \leq {}_nL_{x,t}$. Because the H-P approach (as well as the CCM approach) does not recognize these constraints, one must take care to make sure the point forecast does not violate them. Because we employ probabilistic uncertainty in the forecasts, in the section "Confidence Intervals and

Boundaries," we discuss boundary issues related to these constraints that apply to probabilistic forecasts. We also discuss both the constraint and boundary issues in the "Evaluation" section.

Transferring Uncertainty

Regarding transferring uncertainty to an H-P forecast of ${}_nL_x$, we follow the approach of Swanson and Tayman (2025a, 2025b). It employs the ARIMA (Auto-Regressive Integrated Moving Average) time series method in conjunction with the work of Espenshade and Tayman (1982), thereby transferring the uncertainty information from the ARIMA method's forecast to the population forecast provided by the CCM or H-P approaches.

Before moving on to a description of the Espenshade-Tayman approach, we clarify our use of the term "confidence interval" with respect to forecast uncertainty. It is more common to use the term "forecast interval" or "prediction interval" in the context of forecasting because a "confidence interval," strictly speaking, applies to a sample (Swanson & Tayman, 2014: 204). However, underlying the approach we describe herein is the concept of a "superpopulation," which, as discussed later, describes a population that is but one sample of the infinity of populations that will result by chance from the same underlying social and economic cause systems (Deming and Stephan, 1941). The concept of viewing a forecast as a sample leads us to use the term "confidence interval" rather than "forecast interval" or "prediction interval."

The uncertainty intervals for the ${}_nL_x$ forecasts are based on ARIMA models that forecast the uncertainty in population density (PYL/land area) for the same horizon years. We use "density" because the Espenshade-Tayman (1982) method for translating uncertainty information does so from an estimated "rate," which in this case is the "rate" of PYL density. Other denominators could be used in developing such a "rate," such as the ratio of PYL to housing units. However, using land area as the denominator provides a virtually constant denominator over time, thereby reducing the effort required to assemble the "rate" data. It also serves as a stabilizing factor for ARIMA use, as it dampens the effects of short-term population fluctuations more effectively than, say, housing units, which can also fluctuate over time and are not always in concert with population fluctuations. We note that if the denominator (i.e., "density") is constant over the forecast horizon, it is equivalent to a denominator of 1.00, which means the population itself can be used (Inoue, 2026). We continue, nonetheless, using density for reasons discussed immediately after the illustration using the population of the world as a whole (Swanson and Tayman, 2025b), which is the possibility that a confidence interval can exceed known boundaries (e.g., ${}_nL_x$ can't be less than zero nor greater than 500,000).

As should be obvious, the data assembled to develop the ARIMA age-specific density forecasts should encompass the base data used to develop the ${}_nL_x$ forecasts themselves. The case study we present meets this condition because the historical annual record of each ${}_nL_x$ value covers the period (1959-2024) during which both the ARIMA models used to generate the age-specific (${}_nL_x$) density forecasts and the H-P ${}_nL_x$ forecasts themselves are based. The land area of Estonia is approximately 42,388 km^2.

The approach we take to generate uncertainty measures follows that of Swanson and Tayman (2025a, 2025b), which employs the ARIMA (Auto-Regressive Integrated Moving Average) time series method in conjunction with work by Espenshade and Tayman (1982), whereby we can transfer the uncertainty information found in the ARIMA method's forecast to the PYL forecast provided by the H-P approach. As described by Smith, Tayman, and Swanson (2013: 172-176), an ARIMA model attempts to uncover the stochastic processes that generate a historical data series. The mechanism of this stochastic process is described—based on the patterns observed in the data series—and that mechanism forms the basis for developing forecasts. At its heart, the

ARIMA time series model is a regression-like forecast method. It was popularized by Box and Jenkins (1976) and has been used to analyze and forecast business, economic, and demographic variables. Examples of its use in demographic forecasting include McNown et al. (1995), Pflaumer (1992), Tayman, Smith, and Lin (2007), and Zakria and Muhammad (2009).

As Smith, Tayman, and Swanson (2013: 172-176) discuss, an ARIMA model attempts to uncover the stochastic processes that generate a historical data series. The mechanism of this stochastic process is described—based on the patterns observed in the data series—and that mechanism forms the basis for developing forecasts. As noted earlier, up to three processes can represent the stochastic mechanism: autoregression, differencing, and moving average.

The autoregressive process has memory, in that it is based on the correlation between each value of a variable and all preceding values. The impact of earlier values is assumed to diminish exponentially over time. The number of preceding values explicitly incorporated into the model determines its "order." For example, in a first-order autoregressive process, the current value depends only on the immediately preceding value. However, the immediately preceding value is also a function of the one before it, which is a function of the one before it, and so forth. Thus, all preceding values influence current values, albeit with a declining impact. In a second-order autoregressive process, the current value is explicitly a function of the two immediately preceding values; again, all preceding values have an indirect impact. The differencing process creates a stationary time series (i.e., one with constant average and variance over time, which, in turn, implies there is no trend in the series). A stationary time series is essential for constructing ARIMA models. When a time series is non-stationary, it can often be converted to a stationary time series by differencing the values. First differences are usually sufficient, but second differences are occasionally required (i.e., differences between differences). Logarithmic and square-root transformations can also convert non-stationary variances to stationary ones. The moving average captures a "shock" to the system or an event with a substantial but short-lived impact on the time series. This impact has a limited duration, and then time series trends return to normal. The order of the moving-average process determines the number of time periods affected by the shock.

The most general ARIMA model is usually written as ARIMA(p, d, q), where p is the autoregressive order, d is the degree of differencing, and q is the moving-average order. (ARIMA models based on time intervals of less than one year may also require a seasonal component.) The first and most subjective step in developing an ARIMA model is to identify the values of p, d, and q. The d-value must be determined first because a stationary series is required to identify the autoregressive and moving-average processes correctly. The value of d is the number of times one has to "difference" the series to achieve stationarity (usually 0 or 1, but occasionally 2 in data with non-linear growth). The p- and q-values are also relatively small (often 0, 1, or 2). Autocorrelation (ACF) and partial autocorrelation (PACF) patterns are used to determine the correct values of p and q. For example, a first-order autoregressive model [ARIMA(1, 0, 0)] is characterized by an ACF that declines quickly and exponentially, and a PACF with a significant spike at lag 1. Once p, d, and q are determined, maximum likelihood estimation is used to estimate the ARIMA model parameters. The final step in the estimation process is model diagnosis. An adequate ARIMA model will have random residuals, no significant values in the ACF, and the smallest possible values for p, d, or q. After a successful diagnosis, the ARIMA model is ready for use.

In closing this description of the ARIMA process, we note that there are alternatives, such as dynamic linear modeling (Sevestre and Trognon, 1996), but we employ ARIMA because of our experience with it and its widespread use.

In terms of our actual results, the autocorrelation (ACF) and partial autocorrelation functions (PACF) were used to determine the correct values of p and q (Brockwell and Davis, 2016: Chapter 3). Each of the ${}_nL_x$ ARIMA models has random residuals and the smallest possible values for p, d, or q, as determined by the Portmanteau Test (Ljung and Box, 1978; NCSS, 2024). Using these criteria, each of the selected ${}_nL_x$ ARIMA models has been deemed adequate. We note that there may be other versions that are also "adequate," and that further refinement of the selection process can be done (e.g., using the augmented Dickey-Fuller test (Dickey and Fuller, 1979) to determine the amount of differencing required to achieve stationarity). We used the ARIMA procedure found in the NCSS Statistical Software System (NCSS, 2024) for this set of tasks. After giving an example of how this approach works, again note that we use Estonian data from the HMD to generate and evaluate ${}_nL_x$ forecasts.

Here is an example of this process using the 2050 world population forecast result produced by Swanson and Tayman (2025b: 4-5).

Let P = projected world population (at time t)

Let D = forecasted world population density obtained from ARIMA at time t, and

Let A = land area of the world (131,821,645 square kilometers).

The 2050 ARIMA density forecast shows 73.02, 76.81, and 80.60 persons per square kilometer, respectively, for the land area of the world as a whole (95% Lower Limit of forecasted D, forecasted D, and 95% Upper Limit of forecasted D, respectively). The relative widths of the Lower and Upper Limits are -0.04938 (73.02 / 76.81 - 1) and 0.04938 80.60 / 76.81 - 1), respectively. The 2050 world population forecast found at International Data Base (IDB) site of the U.S. Census Bureau is 9.7 billion (https://www.census.gov/data-tools/demo/idb/#/table?COUNTRY_YEAR=2024&COUNTRY_YR_ANIM=2024&menu=tableViz. Multiplying 9.75 billion by -0.04938 and adding this product to 9.75 billion yields 9.27 billion, the 95% Lower Limit, and adding the product 9.75 billion × 0.04938 to 9.7 billion yields 10.23 billion, the 95% Upper Limit of the 2050 world population forecast found at IDB. Putting it all together, we can say that one can be 95% certain that the 2050 world forecast from IDB lies between 9.27 billion and 10.23 billion.

The Espenshade-Tayman method is based on the idea that a sample is drawn from a population of interest. In this case, the ARIMA results represent the sample, and the CCM forecasts represent the population. This interpretation is derived from the idea of a "super-population" (Hartley and Sielken, 1975; Sampath, 2005; Swanson and Tayman, 2012, pp. 32–33). This concept can be traced back to Deming and Stephan (1941), who observed that even a complete census, for scientific generalizations, describes a population that is but one of the infinity of populations that will result by chance from the same underlying social and economic cause systems. It is a theoretical concept we use to simplify the application of statistical uncertainty to a population forecast, which is considered a statistical model in this context. This approach is conceptually and mathematically different from the classical frequentist theory of finite population sampling (Hartley and Sielken, 1975), but, as pointed out by Ding, Li, and Miratrix (2017), in practice, these two approaches yield identical variance estimators. As such, we believe that this approach is on solid statistical ground. Before moving on, we also note that using the Espenshade-Tayman method (1982) here is not new. In addition to being employed by

Espenshade and Tayman (1982), it has been used by Swanson (1989), Roe, Swanson, and Carlson (1992), and Swanson and Tayman (2025a, 2025b) in demographic applications.

Confidence Interval Boundary Issues

When considering the use of confidence intervals to assess uncertainty in a forecast of ${}_nL_x$ values, there are boundaries to keep in mind. They are related to the constraints but are sufficiently different to warrant discussion. To begin, confidence boundaries that exceed natural or physical boundaries is a known problem, and work has been done to adjust confidence limits when they exceed boundaries (Hazra, 2017; Wu and Neale, 2012). The simplest approach is to truncate the generated confidence interval at the boundary. For example, if the interval is negative, simply truncate it at zero. A more elegant approach is to use a logarithmic transformation, which is suitable because the Espenshade-Tayman approach is based on a ratio (population density). This transformation serves to "normalize" the data before calculating the confidence intervals of the ratio, which can then be transformed back into the original scale when these intervals are transferred onto the forecast population. Another approach is to decrease the width of the confidence interval (e.g., from 95% to 66%) by using a smaller z value (e.g., going from 1.96 to 1.0).

Evaluation of ${}_nL_x$ Forecast for Estonia

The evaluations utilize annual ${}_nL_x$ data taken from the full HMD set for Estonia (1959 to 2024). The evaluations of the point and interval forecasts for 2010-2020 were launched from 2005 using 2005/2000 CCRs, which are held constant over the forecast horizon (see Table 1). Regarding constraint violations in the 2005/2000 H-P model's forecasts, there were two that exceeded 500,000: ${}_5L_0$ for 2015 and 2020. Our (limited) experience with constraint violations led us to conclude that, in this case, they typically appear initially in ${}_5L_0$ and, if left unattended, cascade into subsequent age groups over time. We found that using a weighted average effectively dealt with this issue: Adj ${}_5L_{0,t+n} = (0.7 \times {}_5L_{0,t}) + (0.3 \times {}_5L_{0,t+n})$, followed by scaling the subsequent ${}_nL_{x,t+n}$ (e.g., constrained ${}_5L_5$ = (unconstrained ${}_5L_5$ / ${}_5L_0$) × (constrained ${}_5L_0$). These adjustments not only aligned the forecasts with the constraints but also preserved both the temporal trends in ${}_nL_x$ and the relationships among ${}_nL_x$ values at a given point in time. When needed, a similar weighted-average adjustment was successfully used in the other four countries for which we developed forecasts (Algeria, Australia, Japan, and U.S.A.). We summarize these results in the Discussion section, where we note that different weighted averages may be required to meet these conditions in ${}_nL_x$ populations elsewhere.

The model parameters and point forecasts are shown in Table 1, which forecasts a 5.1% increase in e_0 from 73.0 in 2005 to 76.8 in 2020. There is a clear direct relationship between age group and the 2005-2020 percentage change in PYL. All percentage changes are positive and are under 1% for ages under 30, under 10% for ages 30-34 to 60-64, and exceed 10% for ages 55 and above. The most rapid changes are in the three oldest age groups, ranging from 20.1% for ages 75-70 to 50.8% for ages 85 and older. These changes make sense and are consistent with expectations for an increasingly older population.

(TABLE 1 ABOUT HERE)

We evaluate the error in these forecasts by comparing them to reported ${}_nL_x$ and e_0 values for the corresponding forecast years. We showing the relative differences between the forecasted and the reported values and compute 4 summary measures of error: (1) MALPE (Mean Algebraic Percent Error), (2) MAPE (Mean Absolute Percent Error), (3) MEDAPE (median absolute percent error), and (4) ID (the Index of Dissimilarity Index, also known as the Index of Misallocation, IOM). MALPE provides a view of bias in that if it is negative, then, on average, the forecasted values are lower than the reported values, while MAPE (Swanson and Tayman, 2012: 268-270) shows the mean percent difference between the forecasted and reported values regardless of whether or not the forecasts were too high or too low. MEDAPE is the median of the absolute error distribution. ID measures the extent to which the forecasted values by age differ from the reported values across age groups. It is interpreted as the percentage of the forecasted values by age that would need to be redistributed to match the reported values by age (Swanson and Tayman, 2012: 273). In assessing these measures of error, we use guidelines found in Smith, Tayman, and Swanson (2013: 348-352) and Swanson and Tayman (2012: 281-286) and define substantive errors as at least +-5% but less than +-10% and extreme errors (outliers) as being ± 10% or more.

All of the ID measures, MAPE, MALPE, and MEDAPE are well below 5%, as are the relative differences in e_0 (see Table 2). Only three extreme errors (outliers) are found across the 18 age groups in the three evaluation points, all of which occur in the oldest age group (${}_{\infty}L_{85}$), -11.0% in 2010, -19.8% in 2015, and -20.6% in 2020. The MEDAPE is smaller than the MAPE, indicating that the absolute values of the error distributions are skewed to the right. The bulk of the errors are negative, meaning that the method tends to under-forecast ${}_{n}L_{x}$. These errors indicate that the method is more likely to produce larger errors in older age groups than in younger ones, and that the size of the error is directly related to age group. These errors are consistent with both the MALPE values and the e_0 differences, which are all negative.

(TABLE 2 ABOUT HERE)

As expected, the individual and summary measures of error for 2015 are larger (disregarding the sign) than those for 2010 for ages 25-29 and older (Swanson and Tayman, 2025a, 2025b). For ages under 25, the errors are roughly the same. A somewhat different pattern emerges for 2020 compared to the errors for 2015. Only the 2020 MEDAPE is larger than its 2015 counterpart, while the other summary measures are roughly the same for both years. The errors are similar in both years for ages under 25; the 2020 errors are higher, as would be expected, for ages 25-29 to 50-54. However, the 2020 errors are lower for ages 55-59 to 75-79, but are larger for ages 85+, and similar in value for ages 80-84. The fact that the model underestimates ${}_nL_x$ in 2020, especially in the older age groups, is somewhat surprising because 2020 was impacted by the COVID-19 pandemic (Bentout et al., 2021; Rigby and Satija, 2023). COVID-19 appeared in Estonia in early 2020 (https://www.err.ee/1082077/valitsus-pikendas-eriolukorda) and peaked in early 2021 when Estonia had the highest relative infection rate in the world (https://www.delfi.ee/artikkel/92823061/graafik-eesti-tousis-suhtelise-nakatumisnaiduga-maailmas-esikohale).

Table 3 contains the upper and lower bounds for the 66% confidence intervals along with a measure of the size of the interval known as the half-width (UL -LL) /2 / Point Forecast).[1] It presents this information by age groups, T_0, and e_0 for the 2010, 2015, and 2020 forecasts. These intervals violate the two conditions for an ${}_nL_x$. First, some upper limit ${}_nL_x$ values exceed 500,000, and the situation worsens as the forecast horizon increases. There are none in the 2010 forecast, three in the 2015 forecast, and 10 in the 2020 forecast. While the decrease in ${}_nL_x$ with increasing age holds at the lower limits, it is violated at the upper limits (data not shown). There are four violations in the 2010 and 2015 forecasts and seven in the 2020 forecasts. We did not attempt to correct these inconsistencies because a proper, workable solution needs to be determined, as noted in the discussion.

(TABLE 3 ABOUT HERE)

Continuing with the interval estimates, the 66% "half-widths" increase over time, as should be the case, since we expect uncertainty to increase as one goes further into the future.[2,3] For example, the average of the half-widths over the age groups increases from 7.5% in 2010 to 10.5% in 2020, an increase of 40%. The other noteworthy pattern is that uncertainty increases monotonically with age group. For example, in 2020 the half-width for ages 0-4 (0.2%) rises consistently to 25.8% for ages 85+. One way to test the performance of confidence intervals in predicting the uncertainty of future changes in mortality is to count the number of counts that fall within the intervals associated with a particular forecast (Smith, Tayman, and Lin, 2007). In terms of the interval estimates, Table 4 shows that these results are consistent with the guidelines in Swanson and Tayman (2014), which state that the 66% CIs should encompass the actual (reported) value at least 66% of the time.

(TABLE 4 ABOUT HERE)

Keeping in mind that Swanson and Tayman (2012: 275) point out that it is not generally possible to produce a population estimate for which all error criteria are simultaneously minimized, we find that the evaluation suggests that the method is slightly biased toward under-estimation of ${}_{n}L_{x}$ but is capable of producing point and interval forecasts that are sufficiently accurate that the method should be considered for use. This conclusion comes with the proviso that evaluations of its performance should also continue, both for populations with mortality patterns similar to Estonia's over the case study period and for populations with different mortality patterns.

An Example Forecast for Estonia.

Because the evaluation data cover a 15-year forecast horizon from the launch year of 2005 (with the launch using 2005/2000 CCRs to forecast ${}_{n}L_{x}$ for 2010, 2015, and 2020), we use the same horizon for the example forecast, which is launched from the most recent data (2024) available in the HMD. The result is a forecast launched from 2024 (using 2024/2019 CCRs to forecast ${}_nL_x$ for 2029, 2034, and 2039, the 15-year target). Unlike the evaluation forecast, all point forecasts are less than 500,000 and required no adjustment. The model parameters and point forecasts are shown in Table 5. Forecasts indicate a 1.3% increase in e_0 from 79.4 in 2024 to 80.5 in 2039, slower than the 5.1% increase shown in the evaluation forecast between 2005 and 2020. This slower growth in life expectancy is consistent with the base period change in both forecasts. Life expectancy increased by 2.8% between 2000 and 2005, compared with 0.8% between 2019 and 2014. There is a clear direct relationship between age group and the 2005-2020 percentage change in PYL. All percentage changes are positive and are under 1% for ages under 65, under 10% for ages 65-69 to 80-84, and exceed 15% for ages 85+.

(TABLE 5 ABOUT HERE)

It may be that the slowdown in increases in life expectancy indicates that Estonia's population is approaching the expiration of the "biological warranty" (Olshansky and Carnes, 2009). That is, Estonia's population is approaching the limits of human longevity. This interpretation is consistent with those on the side of the longevity debate who argue that continued increases in human longevity are not likely (see, e.g., Olshansky and Carnes, 2009; Swanson and Sanford, 2012) as opposed to those on the other side who argue that we can expect continued increases (see, e.g., de Gray, 2002; Kurzweil, 2005; Oeppen and Vaupel, 2002). Importantly, unlike the 2020 results reported in the evaluation section, the COVID-19 pandemic affected neither the 2024/2019 CCR model nor its results. Moreover, this slowdown is consistent with other studies of decreasing gains in life expectancy in Europe and other “developed” countries (Abrams et al., 2026; Raleigh, 2019; Rogers et al., 2022; Steel et al., 2025; Swanson et al., 2022).

Table 6 contains the upper and lower bounds for the 66% confidence intervals along with their half-widths. It presents this information by age groups, T_0, and e_0 for the 2029, 2034, and 2039 forecasts. These intervals violate the two conditions for an ${}_nL_x$. First, many upper limit ${}_nL_x$ values exceed 500,000, and the situation worsens as the forecast horizon increases. There are 13 in the 2029 forecast, 14 in the 2034 forecast, and 15 in the 2039 forecast. While the decrease in ${}_nL_x$ with increasing age group holds true for the lower limits, it is violated at the upper limits (data not shown). There are nine in total across the three forecast years. As noted earlier, we did not attempt to correct these inconsistencies at this point in time, but a proper and workable solution needs to be determined.

(TABLE 6 ABOUT HERE)

Continuing with the interval estimates, the 66% "half-widths" increase over time, as should be the case, since we expect uncertainty to increase as one goes further into the future.[4] For example, the average of the half-widths over the age groups increases from 12.5% in 2029 to 14.6% in 2039, an increase of 16%. The other noteworthy pattern is that uncertainty increases monotonically with age group. For example, in 2020 the half-width for ages 0-4 (0.2%) rises consistently to 28.8% for ages 85+.

Discussion

As observed by Tóth (2021: 129), the efficiency of a given mortality forecasting approach largely depends on the character of the given time series, which explains variation in the usefulness of models with different demographic backgrounds. This observation applies not only to Estonia but also to all other ${}_nL_x$ data sets in HMD, which represent 41 countries deemed to have high-quality mortality data. As a member of the UN's "Europe" region, Estonia can be viewed as a representative example of the region, one with low fertility and mortality, but higher levels of migration. Given the ex post facto evaluation for Estonia and its small population, our results suggest that the method will likely work elsewhere.

As support for this statement, we developed point ${}_nL_x$ forecasts for Algeria, Australia, Japan, and the United States. No constraint violations were found in any of the forecasts. The Algerian forecasts by sex were launched from 2011 using 2011/2006 CCRS and evaluated in 2016 and 2021. All of the accuracy summary statistics (MALPE, MAPE, MEDAPE, and ID) were well below 5% in both 2016 and 2021 for both sexes. Regarding Australia, the ${}_nL_x$ forecasts were launched from 2005 using 2005/2000 CCRs and evaluated in 2010, 2015, and 2020. All of the accuracy summary statistics were well below 5% in all three evaluation years. The forecasts for both Japan and the U.S.A. were also launched from 2005 using 2005/2000 CCRs and evaluated in 2010, 2015, and 2020. Again, all accuracy summary statistics were well below 5% across all three evaluation years. Here, again, we note that the COVID-19 pandemic (approximately January 2020 to May 2023) may have affected some of our results (Bentout et al., 2021; Rigby and Satija, 2023).

Constructing CCRs from two consecutive period life tables implies that the two life tables also represent cohort mortality. For example, in the 2000 and 2005 period life tables used to construct the CCRs for the evaluation, ${}_5L_{24,t}$ in 2020 is viewed as the cohort that five years earlier was five years younger, ${}_5L_{20,t-5}$. Given this, none of the CCRs (as opposed to the ratios used for ${}_5L_0$) should exceed 1.00. However, as can be seen in Table 1, the CCRs slightly exceed 1.00 for three age groups (${}_5L_{5,t} / {}_5L_{0,t-5}$, ${}_5L_{10,t} / {}_5L_{5,t-5}$, and ${}_5L_{15,t} / {}_5L_{10,t-5}$). Because period life tables are not constructed with cohorts in mind, these "anomalies" can occur when two successive period life tables are viewed as sets of cohorts. This view serves to remind us that the CCRs generated from two successive period life tables, which is the case in this approach to forecasting $_nL_x$, the CCRs represent approximations of the mortality experience of different sets of cohorts (e.g., in the period life table at time = t+5, $_5L_{10}$ is part of the cohort $_5L_0$ found at time = t-5 as is $_5L_5$ found in the period life table at time = t; whereas in the period life table at time = t+5, $_5L_{15}$ is part of the cohort $_5L_0$ found at time = t-10 as is $_5L_{10}$ found in the period life table at time = t; and so on). If the approximations are close, so that the entire set of CCRs approximates the mortality experience of these sets of cohorts, as appears to be the case with the CCRs in Table 5 for the years 2109 and 2024, the approach should work reasonably well over 20 years, as our evaluation indicates.

If the CCRs do not approximate the cohort mortality experience, one can expect more violations of the constraints, which would require more adjustments than we needed or lead one to the decision not to use this approach if the violations are extensive and pronounced. Regarding the constraints and the simple adjustment we used to overcome violations of them, it may be that they do not work as well in other populations, especially those with different demographic backgrounds. Our analysis showed that the $_nL_x$ point forecasts for the evaluation years had relatively few instances in which the constraints were violated and required minor adjustments. Whereas the violations were more frequent in the probabilistic intervals, most notably in the upper limits, for the $_nL_x$, and tended to increase with the length of the forecast horizon. Correcting these kinds of errors is complicated by the 2-dimensional nature of the violations that must be addressed conjointly and require further study.

One approach that may be useful is the "floors and ceilings" discussion found in Swanson, Schlottmann, and Schmidt (2010). There are also many related tools available online that may help overcome these violations, such as *DemoTools* (https://timriffe.github.io/DemoTools/index.html) and the *Applied Demography Toolbox* (https://applieddemogtoolbox.github.io/). Another avenue of study could focus on the process used to estimate the ARIMA models, which assumed that the equations for each ${}_nL_x$ were unrelated and ignored any correlation and connection between age groups. Perhaps testing a process similar to that for estimating seemingly unrelated regressions (Zellner, 1962) applied to ARIMA models (e.g., Ajobo, Alaba, and Zaenal, 2024; Casals, Garcia-Hiernaux, and Jerez, 2012).

Unlike the methods proposed by, among others, Andersson and Lindholm (2026), Pascariu (2018), and Vaupel (2019), on the one hand, and, on the other, the Lee-Carter Method (and its variants), our new method is directly linked to the fundamental demographic equation, the cornerstone of demographic theory ((Swanson and Tedrow, 2012), an important consideration in developing accurate forecasts (Swanson et al., 2024). From the perspective of formal demography, the H-P approach to forecasting ${}_nL_x$ is a means of forecasting the age structure and size of the stationary population that is associated with a given population at a given point in time. And while a stationary population is not subject to migration, it is subject to mortality and a fixed number of births, which makes it a special case of a "closed" population.

Returning to the point made earlier by Swanson et al. (2023), namely, the closer one comes to having accurate data embedded in a method that is grounded in demographic theory, the more accurate a population forecast method will likely be. This conclusion suggests that this paper can be viewed as a contribution to formal demography, similar to contributions that demonstrated that the H-P approach can be used to take a given population to stability (Swanson, 2024; Swanson, Tedrow, and Baker, 2016). Viewed in this light, the expectation underlying such a forecast is that age-at-death variance will continue even if those who argue that we can expect substantial improvements leading to higher longevity levels (e.g., de Gray, 2002; Kurzweil, 2005; Oeppen and Vaupel, 2002). As Swanson and Tedrow (2021) point out, to achieve zero variance in age at death, all members of each birth cohort would have to die at the same time, which is so unlikely as to be impossible in most, if not all, species, including humans. Thus, given the other qualifications we have discussed, one could expect that the H-P approach to forecasting ${}_nL_x$ would work reasonably well in the face of dramatic improvements in human longevity.

Because it forecasts "years lived," this new approach directly yields life expectancy (by age) by summing ${}_nL_x$ values from 85+ back to age group 0-4. This feature is not found in the Lee-Carter Method and its variants, which forecast age-specific death rates, which in turn require a method to construct a life table from the forecasted ASDR's (e.g., Fergany's method (Fergany, 1971) and the Keyfitz-Frauenthal method (Kintner, 2004: 314-315)). Also, unlike the Lee-Carter Method, which requires input in the form of single years of age over several points in time, the new method has minimal input requirements: A set of age groups at two points in time for which the width of the age groups matches the number of years forming the interval between the two sets. Appendix A contains an example of a life table constructed from the point forecast of ${}_nL_x$ and a probabilistic life expectancy forecast by age using the uncertainty in the ${}_nL_x$.

Some may argue that the use of a simple forecasting method, such as the one we employ here, lacks "real world" predictive ability. To such an argument we reply that Green and Armstrong (2015) find that while no evidence shows complexity improves accuracy, complexity remains popular among (1) researchers because they are rewarded for publishing in highly ranked journals, which favor complexity; (2) methodologists, because complex methods can be used to provide information that supports decision makers' plans; and (3) clients, who may be reassured by incomprehensibility. Regarding our simple forecasting method being "extrapolative," we note that virtually all "objective" forecasting methods not only include elements of judgement, but are in essence extrapolative and based on historical data, to include ARIMA (Box and Jenkins, 1976; Pflaumer, 1992), the Cohort Component Method (Smith, Tayman, and Swanson, 2013: 45-50); structural models (Smith, Tayman, and Swanson, 2013: 215-238), the Lee-Carter mortality forecasting method (Lee and Carter, 1992; Basellini, Camarda, and Booth, 2023), and even what many would consider to be a "subjective" method – The Delphi Technique (Dalkey, 1969). Moreover, while forecasting comes with uncertainty, as Anatole Romaniuc (2010: 14) observed, "Uncertainty should not be a deterrent to exploring the future."

While e_0 has been criticized not only as an indicator of population health (e.g., Crimmins, 2015) but also as an indicator of life duration (e.g., Modig, Rau, and Ahlbom, 2020; Olshansky and Carnes, 2009, 2019), it remains widely used (e.g., Canudus-Romo, 2010; Modig, Rau, and Ahlbom, 2020). Notably, one of its major criticisms is that it fails to address future life expectancy, a criticism that the approach described here has the potential to overcome from both a deterministic and a probabilistic perspective.

It would also be useful to conduct the same type of evaluation we have presented here. In terms of the HMD, this could be done by region of the world (as specified by the United Nations: Africa, the Americas, Asia, Europe, and Oceania). For example, countries other than Algeria (Africa), Australia (Oceania), Japan (Asia), Estonia (Europe), and the United States. Countries in these same regions not found in HMD may be found in the Human Life-Table Database (HLD). Among HLD's 142 countries, there is a fair contingent from Africa, including Botswana, Cameroon, Egypt, The Gambia, Ghana, South Africa, Tanzania, and Zambia. Examples of other UN regions found in HLD include Indonesia, a member of the UN's "Oceania" region, Argentina, a member of the UN's "Americas" region, and India, a member of the UN's "Asia" region. Another area for future research is to conduct evaluations similar to those employed here, focusing on ${}_nL_x$ by gender. Most life tables are gender-specific, and this would be a natural next step for future research. Finally, as noted earlier, further examination is needed of the issues underlying the boundary violations, especially in the upper limits of ${}_nL_x$ confidence intervals.

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Endnote

1. There are two ways in which the ${}_nL_x$ confidence intervals can be used to place confidence intervals around a given T_0 and, subsequently, around e_0. The informal approach (Swanson and Tayman, 2014) would generate confidence intervals for T_0 by adding the LLs and ULs obtained for the ${}_nL_x$ values. The formal approach is called the "error propagation method" (Deming, 1950: 127- 134) and, in different forms, has been used by Alho and Spencer (2005) and Espenshade and Tayman (1982), among others. This approach involves estimating the standard error of the total forecast. This standard error is then multiplied by the total forecast (the sum of the point forecasts) to obtain the margin of error. The margin of error is added to and subtracted from the total forecast to obtain the interval associated with the desired level of confidence (66%, 95%, 99%). Swanson and Tayman (2014) report that both the informal and formal approaches generated virtually indistinguishable confidence intervals when aggregated from the "bottom-up" forecasts to the total forecast. As such, we employed the informal approach here.
2. The half-widths for T_0 and e_0 are the same because e_0 is computed by dividing T_0 by a constant of 100,000.
3. The half-widths for ages under 20 increase slightly when the data are shown with more decimal places.
4. The half-widths for ages under 15 increase slightly when the data are shown with more decimal places.

Table 1. Hamilton-Perry Evaluation Forecasts, Person Years Lived, Estonia, 2010-2020

	$_nL_X$		CCR[a]	$_nL_X$ Point Forecast			Pct. Chg.
Age Group	**2000**	**2005**	**2005/2000**	**2010**	**2015**	**2020**	**2005-2020**
0-4	495,239	496,903	1.003360	498,573	498,669	499,172	0.5%
5-9	493,945	495,991	1.001518	497,658	497,754	498,256	0.5%
10-14	493,226	495,393	1.002932	497,445	497,542	498,043	0.5%
15-19	492,183	494,501	1.002585	496,674	497,157	497,658	0.6%
20-24	489,316	492,104	0.999839	494,422	495,027	495,912	0.8%
25-29	485,818	488,625	0.998588	491,409	492,166	493,169	0.9%
30-34	481,540	484,818	0.997942	487,619	488,850	490,001	1.1%
35-39	475,408	480,016	0.996835	483,284	484,542	486,161	1.3%
40-44	465,412	472,441	0.993759	477,020	478,752	480,390	1.7%
45-49	449,868	460,489	0.989422	467,444	470,485	472,578	2.6%
50-54	429,409	441,805	0.982077	452,236	457,617	460,970	4.3%
55-59	403,415	417,639	0.972590	429,695	438,452	444,031	6.3%
60-64	370,022	387,411	0.960329	401,071	411,347	420,071	8.4%
65-69	328,973	348,749	0.942509	365,138	376,820	386,789	10.9%
70-74	276,452	300,849	0.914510	318,934	332,869	343,798	14.3%
75-79	213,398	240,189	0.868827	261,386	276,225	288,528	20.1%
80-84	144,657	169,435	0.793986	190,707	206,882	218,804	29.1%
85+	113,494	135,368	0.524375	159,831	183,233	204,087	50.8%
T_0[b]	**7,101,775**	**7,302,726**		**7,470,544**	**7,584,390**	**7,678,418**	
e_0[c]	**71.0**	**73.0**		**74.7**	**75.8**	**76.8**	**5.1%**

Source: 2000 and 2005 Human Mortality Database (https://www.mortality.org/Home/Index)

[a] Ages 0-4 CCR = $_4 2005Pop_0$ / $_4 2000Pop_0$
Ages 5-9 to 80-84 CCR = $_n 2005Pop_x$ / $_n 2000Pop_{x-5}$
Ages 85 CCR = $2005Pop_{85+}$ / $2000Pop_{80+}$

[b] Total person years lived (sum of $_nL_x$ over the age groups)

[c] Life expectancy at birth (T_0 / 100,000)

Table 2. Forecast Error Hamilton-Perry Point Forecasts, Person Years Lived, Estonia, 2010-2020

	Algebraic Percent Errors		
Age Group	**2010**	**2015**	**2020**
0-4	0.1%	< 0.1%	< 0.1%
5-9	0.1%	-0.1%	-0.1%
10-14	0.1%	-0.1%	-0.1%
15-19	0.1%	< 0.1%	-0.1%
20-24	0.0%	-0.1%	-0.3%
25-29	-0.1%	-0.4%	-0.6%
30-34	-0.2%	-0.5%	-0.9%
35-39	-0.3%	-0.5%	-1.1%
40-44	-0.6%	-0.8%	-1.5%
45-49	-1.0%	-1.2%	-1.8%
50-54	-1.6%	-1.8%	-2.2%
55-59	-2.6%	-2.7%	-2.6%
60-64	-3.3%	-3.7%	-3.0%
65-69	-3.8%	-4.6%	-3.6%
70-74	-5.0%	-5.7%	-4.7%
75-79	-5.4%	-7.6%	-6.7%
80-84	-5.3%	-9.8%	-9.9%
85+	-11.0%	-19.8%	-20.6%
e_0[a]	**-1.5%**	**-2.3%**	**-2.5%**
	Summary Error Measures[b]		
MALPE	-2.2%	-3.3%	-3.3%
MAPE	2.2%	3.3%	3.3%
MEDAPE	0.8%	1.0%	1.6%
Index of Dissimilarity	0.9%	1.3%	1.2%

[a] Life expectancy at birth

[b] Across all age groups

Table 3. Probabilistic Forecast 66% Intervals, Person Years Lived, Estonia, 2010-2020

	2010		2015		2020	
Age Group	Lower Limit	Upper Limit	Lower Limit	Upper Limit	Lower Limit	Upper Limit
0-4	497,743	499,402	497,833	499,506	498,328	500,017
5-9	496,728	498,587	496,819	498,690	497,314	499,198
10-14	496,497	498,393	496,584	498,499	496,947	499,009
15-19	494,943	498,404	495,191	499,124	495,483	499,834
20-24	491,107	497,736	491,127	498,928	491,503	500,321
25-29	483,125	499,630	482,210	502,054	481,785	504,480
30-34	479,399	495,840	478,962	498,738	478,690	501,312
35-39	470,618	495,949	469,202	499,883	468,528	503,794
40-44	457,656	496,385	455,181	502,323	453,221	507,558
45-49	439,143	495,744	435,781	505,189	432,396	512,759
50-54	418,076	486,395	415,332	499,903	411,689	510,251
55-59	386,870	472,521	384,762	492,142	379,873	507,198
60-64	338,586	463,555	332,748	489,946	326,819	513,323
65-69	312,879	417,398	310,054	443,586	306,543	467,035
70-74	268,490	369,379	267,857	397,881	265,337	422,258
75-79	218,461	304,311	220,938	331,511	221,839	355,216
80-84	155,984	225,429	161,973	251,791	165,088	272,521
85+	127,273	192,389	140,189	226,278	151,408	256,766
T_o[a]	**7,033,577**	**7,907,447**	**7,032,743**	**8,135,970**	**7,022,793**	**8,332,850**
e_o[b]	***70.3***	***79.1***	***70.3***	***81.4***	***70.2***	***83.3***

Table 3 Continued

		Half-Width[c]	
Age Group	2010	2015	2020
0-4	0.2%	0.2%	0.2%
5-9	0.2%	0.2%	0.2%
10-14	0.2%	0.2%	0.2%
15-19	0.3%	0.4%	0.4%
20-24	0.7%	0.8%	0.9%
25-29	1.7%	2.0%	2.3%
30-34	1.7%	2.0%	2.3%
35-39	2.6%	3.2%	3.6%
40-44	4.1%	4.9%	5.7%
45-49	6.1%	7.4%	8.5%
50-54	7.6%	9.2%	10.7%
55-59	10.0%	12.2%	14.3%
60-64	15.6%	19.1%	22.2%
65-69	14.3%	17.7%	20.7%
70-74	15.8%	19.5%	22.8%
75-79	16.4%	20.0%	23.1%
80-84	18.2%	21.7%	24.5%
85+	20.4%	23.5%	25.8%
T_o	**5.8%**	**7.3%**	**8.5%**
Average[d]	**7.5%**	**9.1%**	**10.5%**
e_o	**5.8%**	**7.3%**	**8.5%**

[a] Total person years lived (sum of ${}_nL_x$ over the age groups)

[b] Life expectancy at birth ($T_{0)}$ / 100,000)

[c] ((Upper Limit - Lower Limit) / 2) / Point Forecast × 100

[d] Across all age groups

Table 4. Reported Person Years Lived Contained in the 66% Confidence Intervals, Estonia, 2010-2020

	Reported[a]			Inside 66% Confidence Limit		
Age Group	**2010**	**2015**	**2020**	**2010**	**2015**	**2020**
0-4	497,993	498,628	499,197	1	1	1
5-9	497,286	498,098	498,899	1	1	1
10-14	496,900	497,822	498,649	1	1	1
15-19	496,188	496,981	498,136	1	1	1
20-24	494,662	495,693	497,255	1	1	1
25-29	492,099	494,185	496,047	1	1	1
30-34	488,644	491,115	494,502	1	1	1
35-39	484,656	487,214	491,704	1	1	1
40-44	479,759	482,740	487,561	1	1	1
45-49	471,944	476,040	481,176	1	1	1
50-54	459,493	466,094	471,305	1	1	1
55-59	441,346	450,436	455,676	1	1	1
60-64	414,826	427,064	433,059	1	1	1
65-69	379,498	394,841	401,131	1	1	1
70-74	335,629	352,823	360,633	1	1	1
75-79	276,177	298,818	309,403	1	1	1
80-84	201,286	229,295	242,884	1	1	1
85+	179,561	228,595	256,986	1	0	0
$\mathbf{e_o}$[a]	**75.9**	**77.7**	**78.7**			
Percent Inside Interval[b]				**100.0%**	**94.4%**	**94.4%**

[a]Source: Human Mortality Database (https://www.mortality.org/Home/Index)

[b] Excludes $\mathbf{e_o}$

Table 5. Hamilton-Perry Forecasts, Person Years Lived, Estonia, 2029-2039

	${}_nL_X$		CCR[a]	${}_nL_X$ Point Forecast			Pct. Chg.
Age Group	**2019**	**2024**	**2024/2019**	**2029**	**2034**	**2039**	**2024-2039**
0-4	499,109	499,245	1.000272	499,381	499,517	499,653	0.1%
5-9	498,656	498,934	0.999649	499,070	499,206	499,342	0.1%
10-14	498,391	498,638	0.999964	498,916	499,052	499,188	0.1%
15-19	497,917	497,954	0.999123	498,201	498,479	498,614	0.1%
20-24	496,825	496,715	0.997586	496,752	496,998	497,275	0.1%
25-29	495,378	495,616	0.997567	495,506	495,543	495,789	0.0%
30-34	493,327	493,933	0.997083	494,170	494,061	494,098	0.0%
35-39	490,713	491,130	0.995547	491,733	491,970	491,861	0.1%
40-44	486,987	487,023	0.992480	487,437	488,036	488,270	0.3%
45-49	480,718	480,855	0.987408	480,891	481,299	481,890	0.2%
50-54	471,203	471,521	0.980868	471,655	471,690	472,091	0.1%
55-59	456,335	458,462	0.972961	458,771	458,902	458,936	0.1%
60-64	435,470	438,654	0.961254	440,699	440,996	441,122	0.6%
65-69	404,363	409,513	0.940393	412,507	414,430	414,710	1.3%
70-74	363,401	371,092	0.917720	375,818	378,566	380,331	2.5%
75-79	310,232	319,331	0.878729	326,089	330,242	332,657	4.2%
80-84	242,813	252,343	0.813401	259,744	265,241	268,619	6.5%
85+	256,712	283,053	0.566644	303,379	319,091	331,108	17.0%
T_0[b]	**7,878,550**	**7,944,012**		**7,990,720**	**8,023,319**	**8,045,554**	
e_0[c]	**78.8**	**79.4**		**79.9**	**80.2**	**80.5**	**1.3%**

Source: 2000 and 2005 Human Mortality Database (https://www.mortality.org/Home/Index)

[a] Ages 0-4 CCR = ${}_4 2024Pop_0$ / ${}_4 2019Pop_0$
Ages 5-9 to 80-84 CCR = ${}_n 20245Pop_x$ / ${}_n 2019Pop_{x-5}$
Ages 85 CCR = $2024Pop_{85+}$ / $2019Pop_{80+}$

[b] Total person years lived

[c] Life expectancy at birth

Table 6. Probabilistic Forecast 66% Intervals, Person Years Lived, Estonia, 2029-2039

	2029		2034		2039	
Age Group	**Lower Limit**	**Upper Limit**	**Lower Limit**	**Upper Limit**	**Lower Limit**	**Upper Limit**
0-4	498,524	500,239	498,653	500,382	498,782	500,525
5-9	498,117	500,023	498,248	500,164	498,379	500,305
10-14	497,934	499,898	498,062	500,042	498,191	500,185
15-19	495,698	500,703	495,814	501,143	495,799	501,430
20-24	491,564	501,940	491,430	502,566	491,354	503,196
25-29	481,914	509,062	480,904	510,139	480,174	511,350
30-34	480,615	507,726	479,465	508,657	478,536	509,659
35-39	470,450	513,016	468,999	514,940	467,344	516,377
40-44	454,419	520,454	452,312	523,760	450,052	526,488
45-49	431,653	530,128	427,899	534,700	424,567	539,214
50-54	410,554	532,757	405,247	538,133	400,550	543,632
55-59	379,107	538,436	371,901	545,904	364,946	552,926
60-64	321,889	559,508	309,230	572,762	296,728	585,515
65-69	306,480	518,535	297,128	531,732	286,784	542,636
70-74	269,925	481,712	261,323	495,808	252,203	508,458
75-79	234,912	417,267	229,891	430,595	223,947	441,367
80-84	185,350	334,139	184,289	346,194	181,309	355,930
85+	216,117	390,642	223,524	414,657	228,771	433,446
T_o[a]	**7,125,221**	**8,856,185**	**7,074,318**	**8,972,277**	**7,018,416**	**9,072,640**
***e_o*[b]**	***71.3***	***88.6***	***70.7***	***89.7***	***70.2***	***90.7***

Table 6. Continued

	Half-Width[c]		
Age Group	**2029**	**2034**	**2039**
0-4	0.2%	0.2%	0.2%
5-9	0.2%	0.2%	0.2%
10-14	0.2%	0.2%	0.2%
15-19	0.5%	0.5%	0.6%
20-24	1.0%	1.1%	1.2%
25-29	2.7%	2.9%	3.1%
30-34	2.7%	3.0%	3.1%
35-39	4.3%	4.7%	5.0%
40-44	6.8%	7.3%	7.8%
45-49	10.2%	11.1%	11.9%
50-54	13.0%	14.1%	15.2%
55-59	17.4%	19.0%	20.5%
60-64	27.0%	29.9%	32.7%
65-69	25.7%	28.3%	30.8%
70-74	28.2%	31.0%	33.7%
75-79	28.0%	30.4%	32.7%
80-84	28.6%	30.5%	32.5%
85+	28.8%	29.9%	30.9%
T_o	**10.8%**	**11.8%**	**12.8%**
Average[d]	**12.5%**	**13.6%**	**14.6%**
e_o	**10.8%**	**11.8%**	**12.8%**

[a] Total person years lived (sum of $_nL_x$ over the age groups)

[b] Life expectancy at birth ($T_{0)}$ / 100,000)

[c] ((Upper Limit - Lower Limit) / 2) / Point Forecast × 100

[d] Across all age groups

Appendix A.

Life Table and Probabilistic Forecasts of Life Expectancy Derived from a Person-Years Lived Forecast

Table A.1. Life Table Derived from Forecasted $_nL_X$ Values, Estonia 2039

Age Group	$_nq_x$	$_nl_x$	$_nd_x$	$_nL_x$	$_nT_x$	$_ne_x$
0-4	0.0003	100,000	32	499,653	8,045,554	80.46
5-9	0.0030	99,968	299	499,342	7,545,901	75.48
10-14	0.0003	99,669	31	499,188	7,046,559	70.70
15-19	0.0011	99,638	114	498,614	6,547,371	65.71
20-24	0.0027	99,524	267	497,275	6,048,757	60.78
25-29	0.0030	99,256	297	495,789	5,551,481	55.93
30-34	0.0034	98,960	338	494,098	5,055,693	51.09
35-39	0.0045	98,622	447	491,861	4,561,595	46.25
40-44	0.0073	98,176	717	488,270	4,069,734	41.45
45-49	0.0131	97,459	1,273	481,890	3,581,464	36.75
50-54	0.0203	96,186	1,956	472,091	3,099,574	32.22
55-59	0.0279	94,230	2,626	458,936	2,627,483	27.88
60-64	0.0388	91,604	3,556	441,122	2,168,547	23.67
65-69	0.0599	88,048	5,272	414,710	1,727,425	19.62
70-74	0.0829	82,776	6,862	380,331	1,312,715	15.86
75-79	0.1253	75,914	9,516	332,657	932,385	12.28
80-84	0.4000	66,398	26,559	268,619	599,728	9.03
85+	1.0000	39,839	39,839	331,108	331,108	8.31

Calculation Steps

Step 1 $_nL_x$ from Hamilton-Perry based forecast (Table 5)

Step 2 $_\infty T_{85} = {}_\infty L_{85}$

$_nT_{80} = {}_\infty L_{85} + {}_nL_{80}$ .... $_nT_0 = {}_nT_5 + {}_nL_0$

Step 3 $_nl_0 = 100{,}000$

$_nl_5 = {}_nL_5 / 5 \times 1.001$

$_nl_{10} = {}_nL_5 / 5 \times (0.999 / 1.001)$ .... $_nl_{80} = nL_{75} / 5 \times (0.999 / 1.001)$

$_nl_{85} = 1.60 \times {}_nl_{80}$

Step 4 $_nd_0 = {}_nl_0 - {}_nl_5$ .... $_nd_{80} = {}_nl_{80} - {}_\infty l_{85}$

Step 5 $_\infty d_{85} = {}_\infty l_{85}$

Step 6 $_ne_x = {}_nT_x / {}_nl_x$

Step 7 $_nq_x = {}_nd_x / {}_nl_x$

Table A.2. Probabilistic Life Expectancy Forecast 66% Intervals, Estonia, 2039

	Person Years Lived[a]			Relative Limits[b]		Life expectancy[c]			
Age Group	Lower	Point	Upper	Lower	Upper	Lower	Point	Upper	Half-Width[d]
0-4	498,782	499,653	500,525			70.18	80.46	90.73	12.8%
5-9	498,379	499,342	500,305	-0.00193	0.00193	75.34	75.48	75.63	0.2%
10-14	498,191	499,188	500,185	-0.00200	0.00200	70.56	70.70	70.84	0.2%
15-19	495,799	498,614	501,430	-0.00565	0.00565	65.34	65.71	66.08	0.6%
20-24	491,354	497,275	503,196	-0.01191	0.01191	60.05	60.78	61.50	1.2%
25-29	480,174	495,789	511,350	-0.03149	0.03139	54.17	55.93	57.69	3.1%
30-34	478,536	494,098	509,659	-0.03149	0.03149	49.48	51.09	52.70	3.1%
35-39	467,344	491,861	516,377	-0.04985	0.04984	43.95	46.25	48.56	5.0%
40-44	450,052	488,270	526,488	-0.07827	0.07827	38.21	41.45	44.70	7.8%
45-49	424,567	481,890	539,214	-0.11896	0.11896	32.38	36.75	41.12	11.9%
50-54	400,550	472,091	543,632	-0.15154	0.15154	27.34	32.22	37.11	15.2%
55-59	364,946	458,936	552,926	-0.20480	0.20480	22.17	27.88	33.59	20.5%
60-64	296,728	441,122	585,515	-0.32733	0.32733	15.92	23.67	31.42	32.7%
65-69	286,784	414,710	542,636	-0.30847	0.30847	13.57	19.62	25.67	30.8%
70-74	252,203	380,331	508,458	-0.33688	0.33688	10.52	15.86	21.20	33.7%
75-79	223,947	332,657	441,367	-0.32679	0.32679	8.27	12.28	16.30	32.7%
80-84	181,309	268,619	355,930	-0.32503	0.32504	6.10	9.03	11.97	32.5%
85+	228,771	331,108	433,446	-0.30908	0.30908	5.74	8.31	10.88	30.9%
T_0	**7,018,416**	**8,045,554**	**9,072,640**						
e_0	**70.18**	**80.46**	**90.73**						

[a] From Hamilton-Perry based forecast Table 6

[b] Proportional differnce between lower and upper ranges and point forecasts of person years lived

[c] Point forecasts from Table A.1.

[d] Upper and lower bounds

Ages 0-4 show the life expectancy at birth intervals calculated in this table

Other age groups limits = Point + (Relative Upper limit or Relative Lower Limit × Point)

[d] ((Upper Limit - Lower Limit) / 2) / Point Forecast × 100